Joint IMD-WMO group fellowship Training On Numerical Weather Prediction
By
Meteorological Training Institute, India Meteorological Department (IMD), Pune

Atmospheric Boundary Layer (ABL) and its Parameterizations – Part III

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19 & 20 October 2021

1. References

1. Turbulence in the Atmosphere – J. C. Wyngaard (Cambridge University Press) *

2. An Introduction to Boundary Layer Meteorology – R. B. Stull (Kluwer Academic Publishers) *

3. An Introduction to Fluid Dynamics – G. K. Batchelor (Cambridge University Press)

1. References

4. Boundary-Layer Theory (7th Edition) – H. Schlichting
(McGraw-Hill Book Company)

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(Oxford University Press)

6. Fluid Mechanics – Kundu and Cohen
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2. Atmospheric Boundary Layer

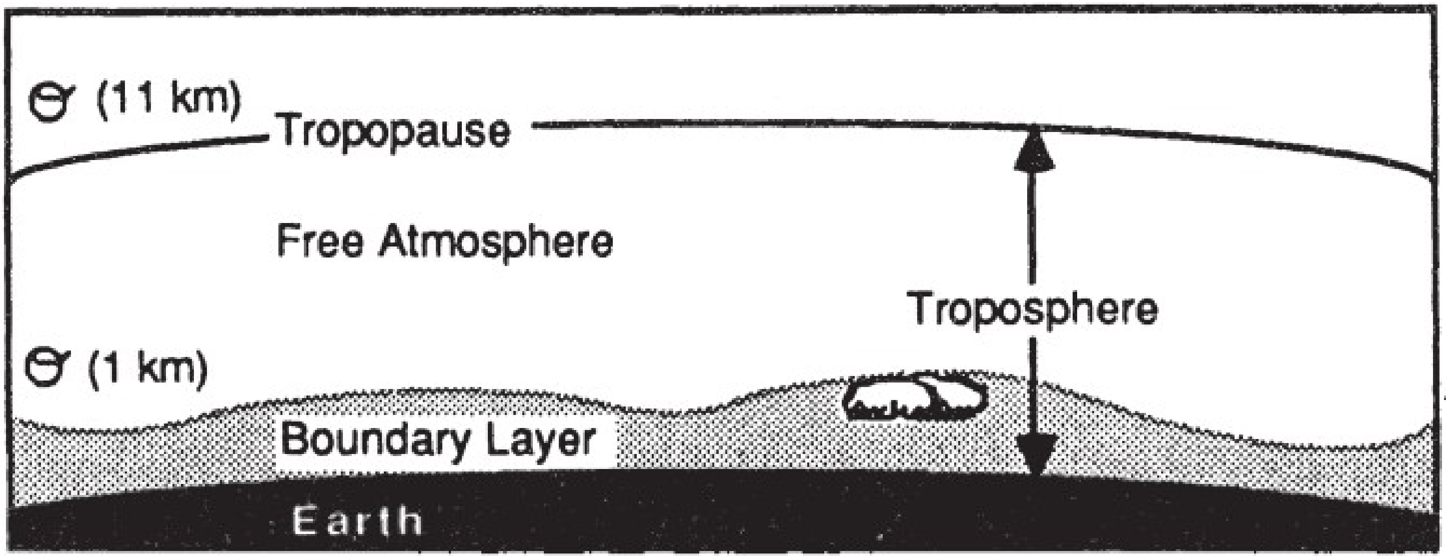
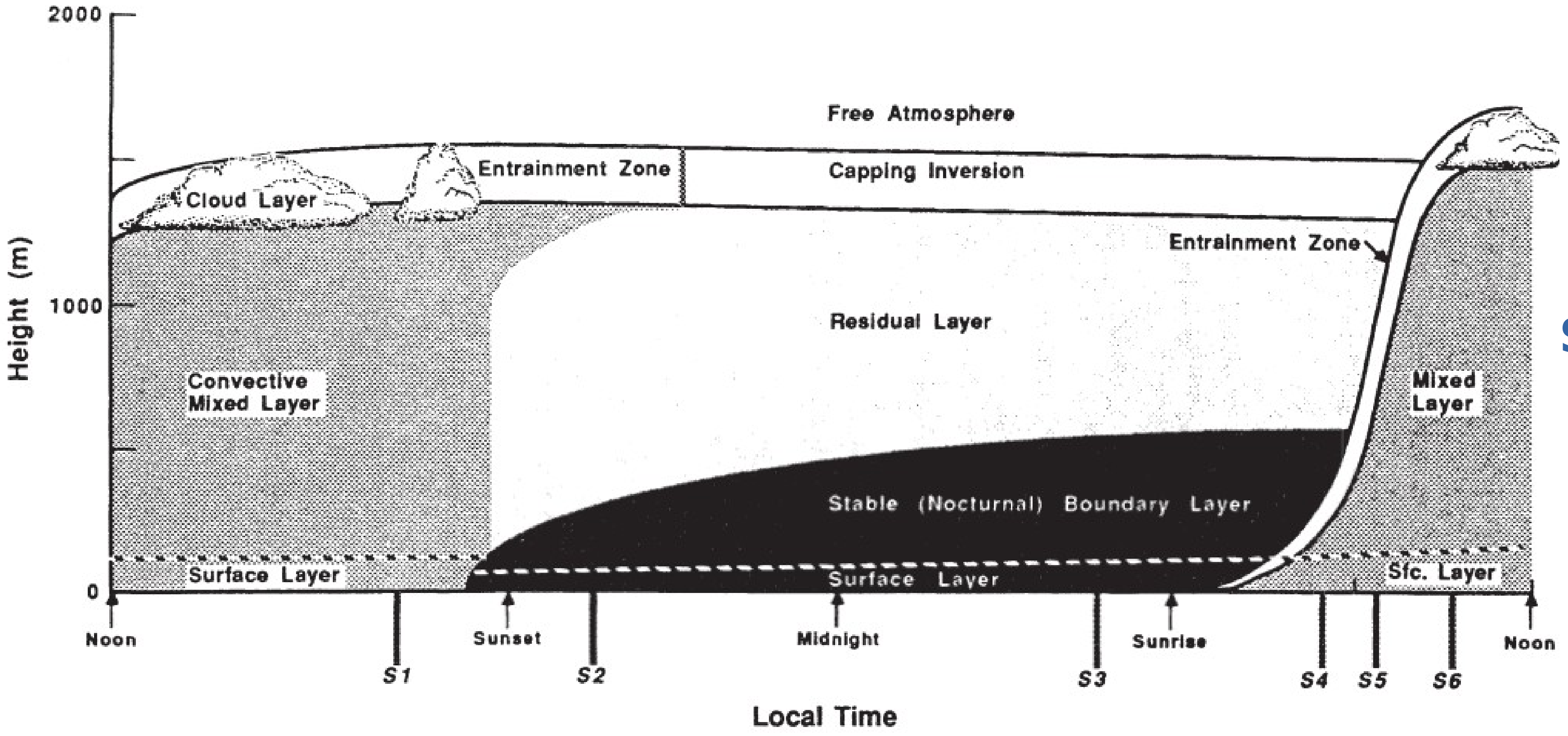


Fig. 1.1 The troposphere can be divided into two parts: a boundary layer (shaded) near the surface and the free atmosphere above it.

Stull

2. Atmospheric Boundary Layer



Stull

2. Atmospheric Boundary Layer

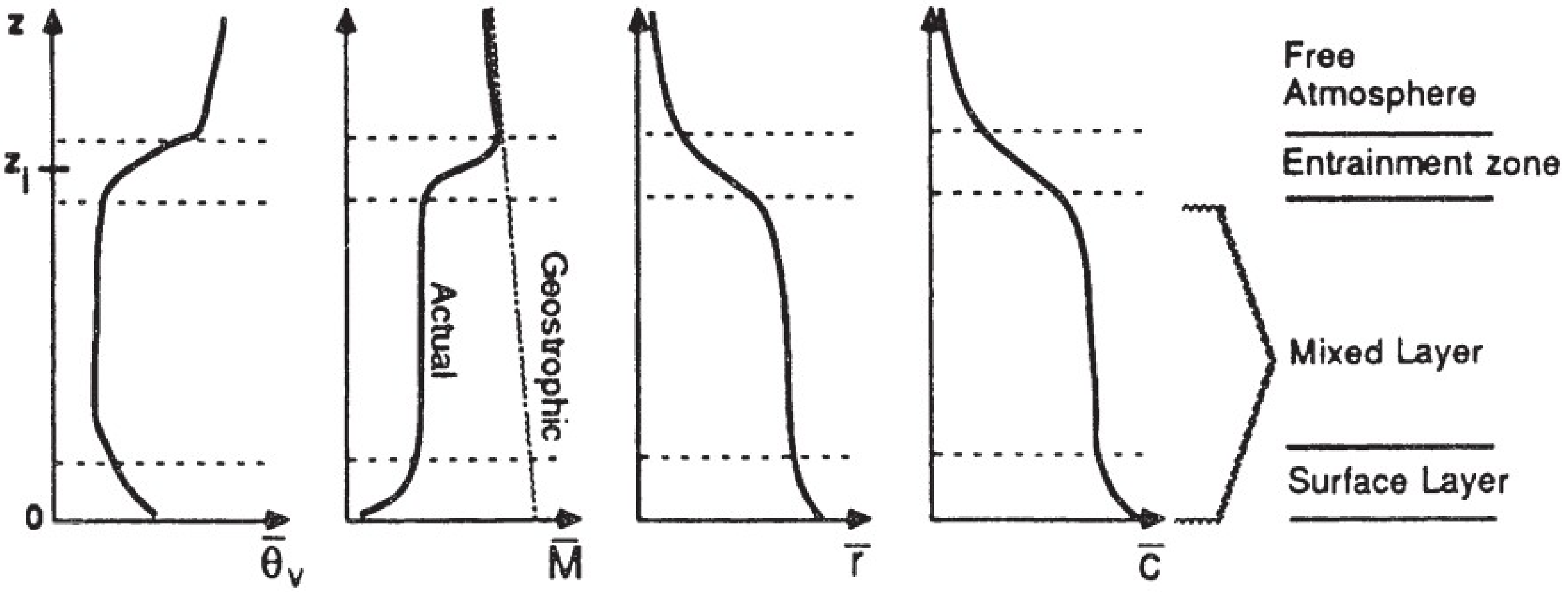


Fig. 1.9 Typical daytime profiles of mean virtual potential temperature $\overline{\theta}_v$, wind speed $\overline{M}$ (where $\overline{M}^2 = \overline{u}^2 + \overline{v}^2$), water vapor mixing ratio $\overline{r}$, and pollutant concentration $\overline{c}$.

Stull

2. Atmospheric Boundary Layer

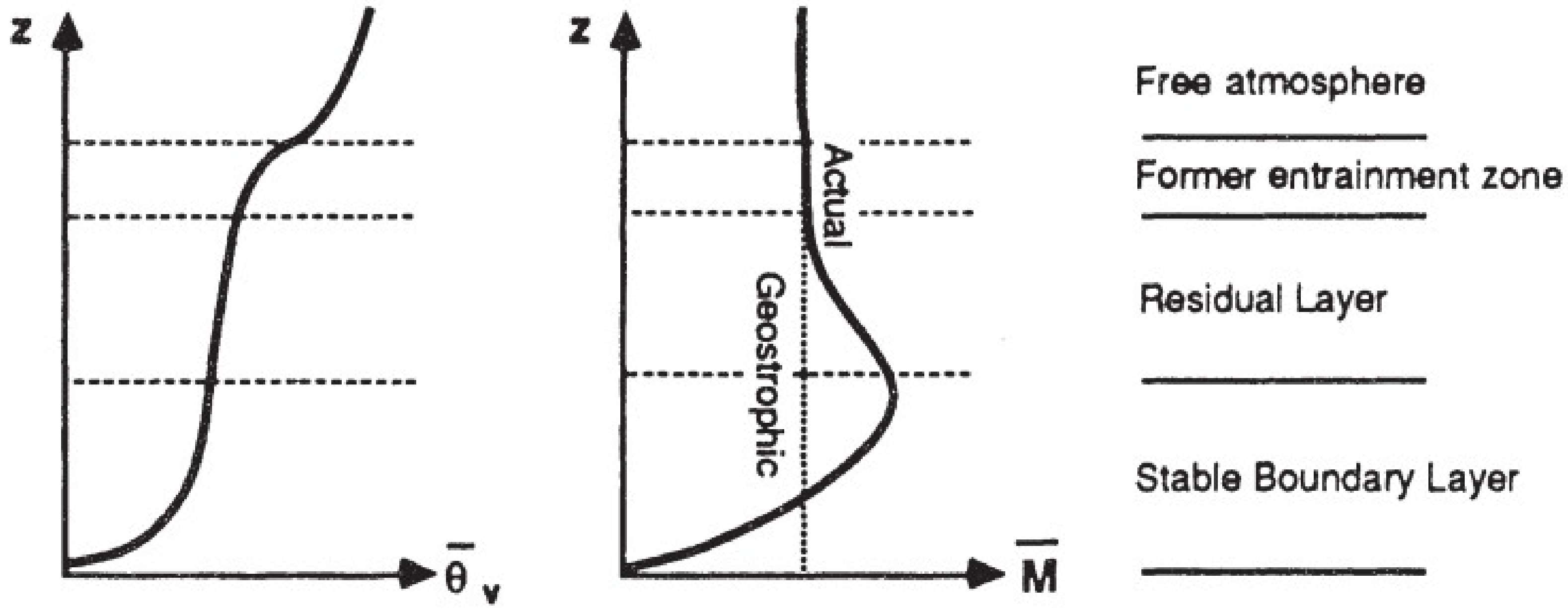
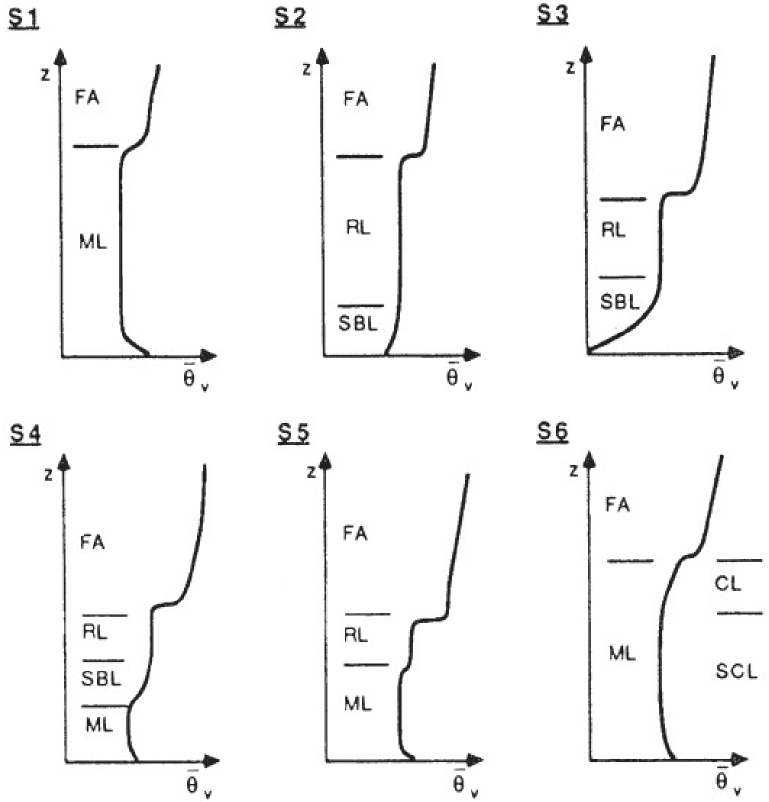


Fig. 1.11 Mean virtual potential temperature, $\overline{\theta}_v$, and wind speed, $\overline{M}$, profiles for an idealized stable boundary layer in a high-pressure region.

Stull

2. Atmospheric Boundary Layer

Fig. 1.12
Profiles of mean virtual potential temperature, $\bar{\theta}_v$, showing the boundary-layer evolution during a diurnal cycle starting at about 1600 local time. S1-S6 identify each sounding with an associated launch time indicated in Fig. 1.7.



Stull

2. Stability Parameters

$$R_f = \frac{\left(\frac{g}{\overline{\theta}_v}\right)\left(\overline{w'\theta_v'}\right)}{\left(\overline{u_i'u_j'}\right)\frac{\partial \overline{U}_i}{\partial x_j}}$$

Flux Richardson Number

2. Stability Parameters

$$R_f = \frac{\left(\frac{g}{\overline{\theta}_v}\right)\overline{(w'\theta_v')}}{\overline{(u_i'u_j')}\frac{\partial \overline{U}_i}{\partial x_j}}$$

Flux Richardson Number

$$R_f = \frac{\left(\frac{g}{\overline{\theta}_v}\right)\overline{(w'\theta_v')}}{\overline{(u'w')}\frac{\partial \overline{U}}{\partial z} + \overline{(v'w')}\frac{\partial \overline{V}}{\partial z}}$$

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Flux Richardson Number

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$$Ri = \frac{\frac{g}{\overline{\theta_v}}\frac{\partial \overline{\theta_v}}{\partial z}}{\left[\left(\frac{\partial \overline{U}}{\partial z}\right)^2 + \left(\frac{\partial \overline{V}}{\partial z}\right)^2\right]}$$

Gradient Richardson Number

2. Stability Parameters

$$R_f = \frac{\left(\frac{g}{\overline{\theta_v}}\right)\overline{(w'\theta_v')}}{\overline{(u_i'u_j')}\frac{\partial \overline{U_i}}{\partial x_j}}$$

Flux Richardson Number

$$R_f = \frac{\left(\frac{g}{\overline{\theta_v}}\right)\overline{(w'\theta_v')}}{\overline{(u'w')}\frac{\partial \overline{U}}{\partial z} + \overline{(v'w')}\frac{\partial \overline{V}}{\partial z}}$$

$$Ri = \frac{\frac{g}{\overline{\theta_v}}\frac{\partial \overline{\theta_v}}{\partial z}}{\left[\left(\frac{\partial \overline{U}}{\partial z}\right)^2 + \left(\frac{\partial \overline{V}}{\partial z}\right)^2\right]}$$

Gradient Richardson Number

Bulk Richardson Number

$$R_B = \frac{g\ \Delta\overline{\theta_v}\ \Delta z}{\overline{\theta_v}\left[(\Delta\overline{U})^2 + (\Delta\overline{V})^2\right]}$$

2. Stability Parameters

$$R_f = \frac{\left(\frac{g}{\overline{\theta_v}}\right)\left(\overline{w'\theta_v'}\right)}{\left(\overline{u_i'u_j'}\right)\frac{\partial \overline{U_i}}{\partial x_j}}$$

Flux Richardson Number

Applied in the Surface Layer

$$\overline{u_i u_j} \sim U_*^2,\ \partial U_i/\partial x_j \sim U_*/\kappa z$$

2. Stability Parameters

$$R_f = \frac{\left(\frac{g}{\overline{\theta_v}}\right)(\overline{w'\theta_v'})}{(\overline{u_i'u_j'})\frac{\partial \overline{U_i}}{\partial x_j}}$$

Flux Richardson Number

Applied in the Surface Layer

$$\overline{u_i u_j} \sim U_*^2,\ \partial U_i/\partial x_j \sim U_*/\kappa z$$

$$\zeta = \frac{z}{L} = \frac{-k\, z\, g\, (\overline{w'\theta_v'})_s}{\overline{\theta}_v\, u_*^3}$$

"L" is the Obukhov length

"z" is the height of the measurement location in the Surface Layer

2. Stability Parameters

$$L = -\frac{\overline{\theta_v} U_*^3}{\kappa g \left(\overline{w'\theta_v'}\right)_s}$$

$$\zeta = \frac{z}{L}$$

$$\zeta < 0, \text{ unstable}$$

$$\zeta > 0, \text{ stable}$$

"L" is the Obukhov length

"z" is the measurement location in the Surface Layer

Surface Layer stability parameter

2. Stability Parameters

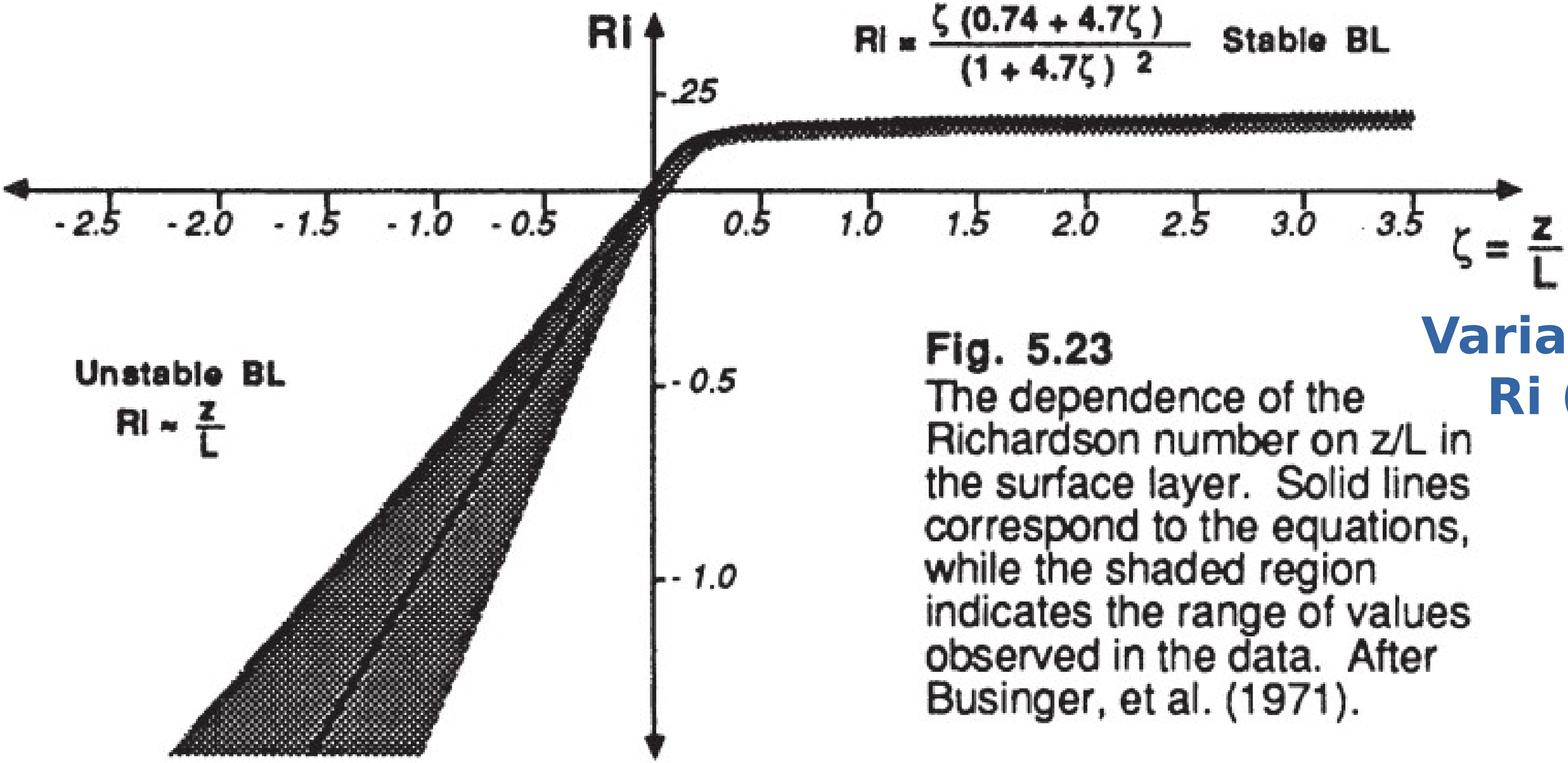

Fig. 5.23
The dependence of the Richardson number on z/L in the surface layer. Solid lines correspond to the equations, while the shaded region indicates the range of values observed in the data. After Businger, et al. (1971).

Variation of z/L with Ri (Gradient Ri)

2. Stability Parameters

$$w_* = \left[\frac{g\, z_i}{\overline{\theta}_v} \left(\overline{w'\theta_v'} \right)_s \right]^{1/3}$$

Convective Velocity Scale

zi is the height of the top of the ML

$\overline{w'\theta_v'} > 0$, convective i.e. heat flux upward from the surface

$$\zeta = \frac{z}{L} = -\frac{k\, z\, w_*^3}{z_i\, u_*^3}$$

Surface Layer stability parameter in terms of convective and shear velocity scales

2. Stability Parameters

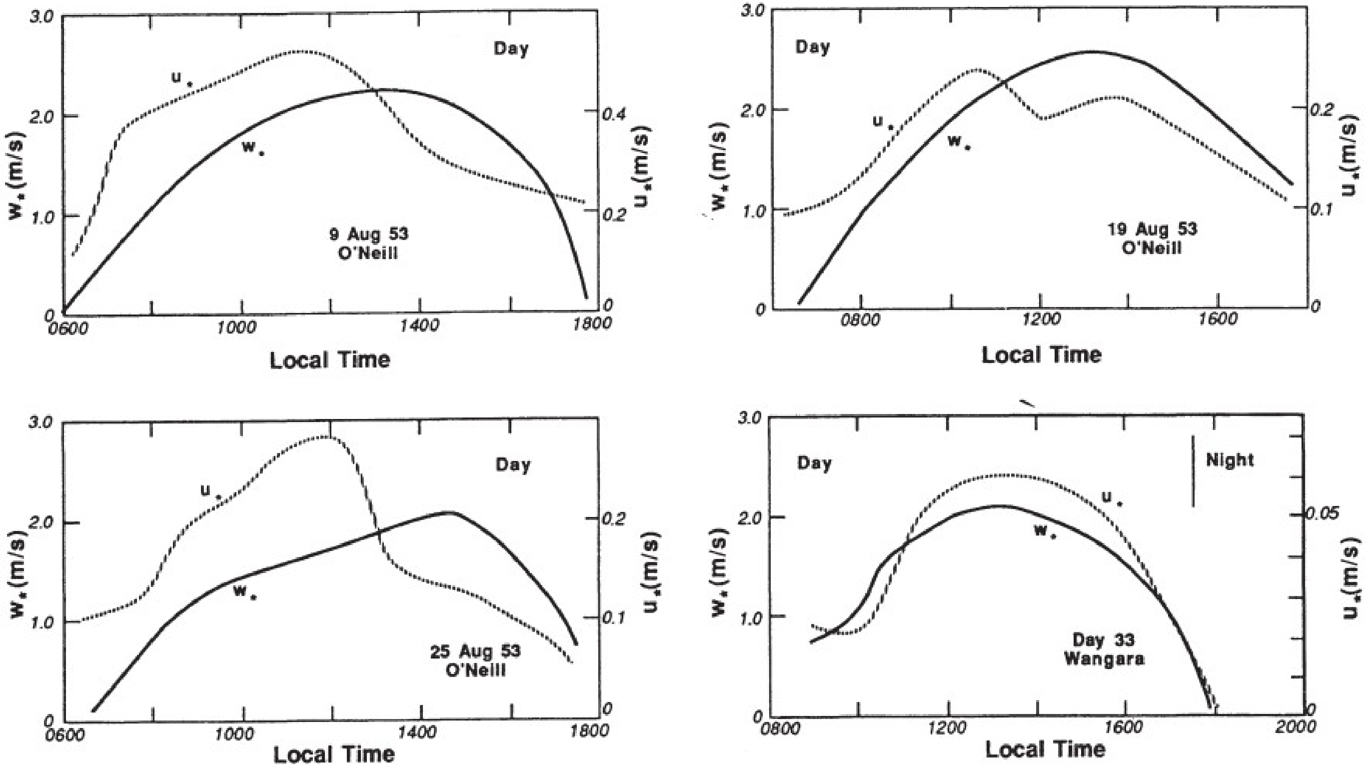

Fig. 4.1 Sample variations of the friction velocity, u_*, and the convective scaling velocity, w_*, with time for the O'Neill, (Nebraska) and Wangara (Australia) field programs.

Stull

Diurnal variation of w* and U*

3. TKE Budget in the ABL

$$\frac{\partial \overline{u_i'^2}}{\partial t} + \overline{U}_j \frac{\partial \overline{u_i'^2}}{\partial x_j} = +2\,\delta_{i3}\,\frac{g\,(\overline{u_i'\theta_v'})}{\overline{\theta}_v} - 2\overline{u_i'u_j'}\,\frac{\partial \overline{U}_i}{\partial x_j} - \frac{\partial\,(\overline{u_j'u_i'^2})}{\partial x_j} - \frac{2}{\overline{\rho}}\,\frac{\partial\,(\overline{u_i'p'})}{\partial x_i} - 2\varepsilon$$

3. TKE Budget in the ABL

$$\frac{\partial \overline{u_i'^2}}{\partial t} + \overline{U}_j \frac{\partial \overline{u_i'^2}}{\partial x_j} = +2\,\delta_{i3}\frac{g\,(\overline{u_i'\theta_v'})}{\overline{\theta}_v} - 2\overline{u_i'u_j'}\frac{\partial \overline{U}_i}{\partial x_j} - \frac{\partial\,(\overline{u_j'u_i'^2})}{\partial x_j} - \frac{2}{\overline{\rho}}\frac{\partial\,(\overline{u_i'p'})}{\partial x_i} - 2\varepsilon$$

$$\frac{\partial \overline{u'^2}}{\partial t} + \overline{U}_j \frac{\partial \overline{u'^2}}{\partial x_j} = -2\overline{u'u_j'}\frac{\partial \overline{U}}{\partial x_j} - \frac{\partial\,(\overline{u_j'u'^2})}{\partial x_j} - \frac{2}{\overline{\rho}}\frac{\partial(\overline{u'p'})}{\partial x} + \overline{\frac{2p'}{\overline{\rho}}\frac{\partial u'}{\partial x}} - 2\nu\overline{\left(\frac{\partial u'}{\partial x_j}\right)^2}$$

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$$\frac{\partial \overline{u_i'^2}}{\partial t} + \overline{U}_j \frac{\partial \overline{u_i'^2}}{\partial x_j} = + 2\,\delta_{i3} \frac{g\,(\overline{u_i'\theta_v'})}{\overline{\theta}_v} - 2\overline{u_i'u_j'}\frac{\partial \overline{U}_i}{\partial x_j} - \frac{\partial\,(\overline{u_j'u_i'^2})}{\partial x_j} - \frac{2}{\overline{\rho}}\frac{\partial\,(\overline{u_i'p'})}{\partial x_i} - 2\varepsilon$$

$$\frac{\partial \overline{u'^2}}{\partial t} + \overline{U}_j \frac{\partial \overline{u'^2}}{\partial x_j} = - 2\overline{u'u_j'}\frac{\partial \overline{U}}{\partial x_j} - \frac{\partial\,(\overline{u_j'u'^2})}{\partial x_j} - \frac{2}{\overline{\rho}}\frac{\partial(\overline{u'p'})}{\partial x} + \frac{\overline{2p'}}{\overline{\rho}}\frac{\overline{\partial u'}}{\partial x} - 2\nu\overline{\left(\frac{\partial u'}{\partial x_j}\right)^2}$$

$$\frac{\partial \overline{v'^2}}{\partial t} + \overline{U}_j \frac{\partial \overline{v'^2}}{\partial x_j} = - 2\overline{v'u_j'}\frac{\partial \overline{V}}{\partial x_j} - \frac{\partial\,(\overline{u_j'v'^2})}{\partial x_j} - \frac{2}{\overline{\rho}}\frac{\partial(\overline{v'p'})}{\partial y} + \frac{\overline{2p'}}{\overline{\rho}}\frac{\overline{\partial v'}}{\partial y} - 2\nu\overline{\left(\frac{\partial v'}{\partial x_j}\right)^2}$$

3. TKE Budget in the ABL

$$\frac{\partial \overline{u_i'^2}}{\partial t} + \overline{U}_j \frac{\partial \overline{u_i'^2}}{\partial x_j} = +2\,\delta_{i3}\frac{g\,(\overline{u_i'\theta_v'})}{\overline{\theta}_v} - 2\overline{u_i'u_j'}\frac{\partial \overline{U}_i}{\partial x_j} - \frac{\partial (\overline{u_j'u_i'^2})}{\partial x_j} - \frac{2}{\overline{\rho}}\frac{\partial (\overline{u_i'p'})}{\partial x_i} - 2\varepsilon$$

$$\frac{\partial \overline{u'^2}}{\partial t} + \overline{U}_j \frac{\partial \overline{u'^2}}{\partial x_j} = -2\overline{u'u_j'}\frac{\partial \overline{U}}{\partial x_j} - \frac{\partial (\overline{u_j'u'^2})}{\partial x_j} - \frac{2}{\overline{\rho}}\frac{\partial (\overline{u'p'})}{\partial x} + \frac{\overline{2p'}}{\overline{\rho}}\frac{\overline{\partial u'}}{\partial x} - 2\nu\overline{\left(\frac{\partial u'}{\partial x_j}\right)^2}$$

$$\frac{\partial \overline{v'^2}}{\partial t} + \overline{U}_j \frac{\partial \overline{v'^2}}{\partial x_j} = -2\overline{v'u_j'}\frac{\partial \overline{V}}{\partial x_j} - \frac{\partial (\overline{u_j'v'^2})}{\partial x_j} - \frac{2}{\overline{\rho}}\frac{\partial (\overline{v'p'})}{\partial y} + \frac{\overline{2p'}}{\overline{\rho}}\frac{\overline{\partial v'}}{\partial y} - 2\nu\overline{\left(\frac{\partial v'}{\partial x_j}\right)^2}$$

$$\frac{\partial \overline{w'^2}}{\partial t} + \overline{U}_j \frac{\partial \overline{w'^2}}{\partial x_j} = \frac{2g(\overline{w'\theta_v'})}{\overline{\theta}_v} - 2\overline{w'u_j'}\frac{\partial \overline{W}}{\partial x_j} - \frac{\partial (\overline{u_j'w'^2})}{\partial x_j} - \frac{2}{\overline{\rho}}\frac{\partial (\overline{w'p'})}{\partial z} + \frac{\overline{2p'}}{\overline{\rho}}\frac{\overline{\partial w'}}{\partial z} - 2\nu\overline{\left(\frac{\partial w'}{\partial x_j}\right)^2}$$

4. Surface Layer Similarity Theory

What is the Surface Layer??

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Close to a horizontally homogeneous surface,

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- **Therefore, the divergences of turbulent momentum and heat fluxes must be negligibly small OR the fluxes must be constant**

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- **Still one is sufficiently away from the surface so that divergences of viscous momentum flux and diffusion heat flux are negligible**
- **Horizontal pressure gradient forces are weak, generally**
- **Therefore, the divergences of turbulent momentum and heat fluxes must be negligibly small OR the fluxes must be constant**
- **Surface Layer is the lowest few tens to hundreds of meter of the ABL where the turbulent momentum and heat fluxes are constant**

4. Surface Layer Similarity Theory

Surface Layer closely corresponds to the Overlap Layer in Laboratory Turbulent Boundary Layers where the turbulent shear stress OR momentum flux is constant with respect to *z*

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Surface Layer

ϕ_M is dimensionless wind shear

$$\frac{\kappa z}{U_*}\frac{\partial U}{\partial z} = \phi_M = f\left(\zeta\right)$$

Proposal by Monin and Obukhov

$f(\zeta = 0) = 1$ recovers the overlap layer scaling

4. Surface Layer Similarity Theory

Dimensionless gradients in the Surface Layer

$$\phi_M = \frac{k\,z}{u_*}\frac{\partial \overline{U}}{\partial z}$$

$$\phi_H = \frac{k\,z}{\theta_*^{SL}}\frac{\partial \overline{\theta}}{\partial z}$$

$$\phi_E = \frac{k\,z}{q_*^{SL}}\frac{\partial \overline{q}}{\partial z}$$

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Different Classes of Similarity Theories

- **Monin-Obukhov (Surface Layer) Similarity**
- **Mixed Layer Similarity**
- **Local Similarity**
- **Free Convection Similarity**
- **Rossby-Number Similarity**

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For Surface Layer Similarity

ϕ_M, ϕ_H, ϕ_E universal functions of ζ

4. Surface Layer Similarity Theory

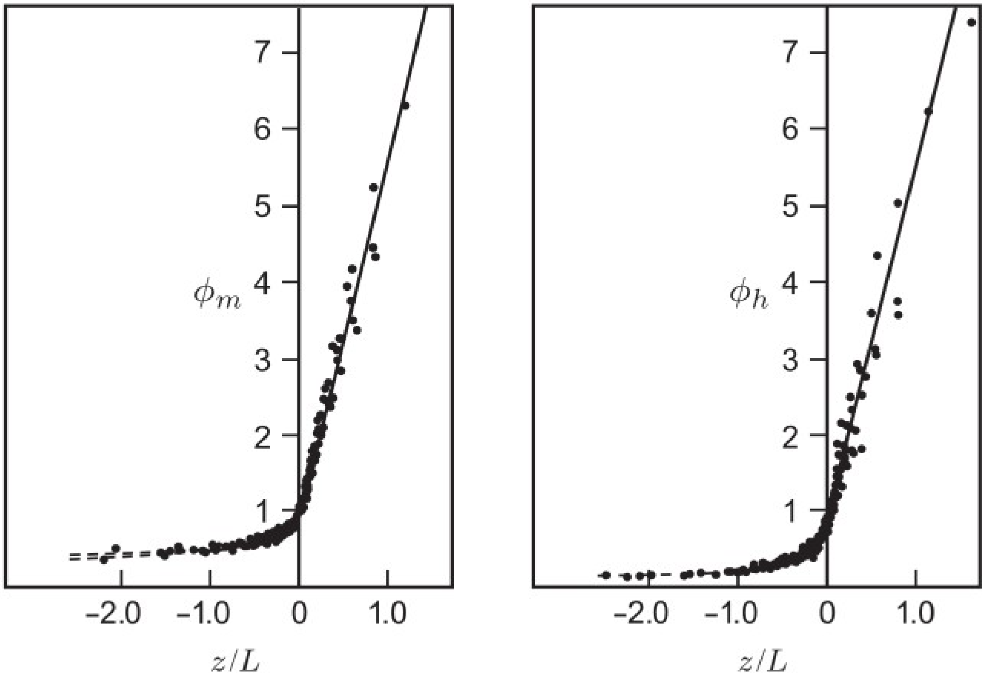

Figure 10.3 The M-O functions for mean wind shear (left) and mean potential temperature gradient (right), Eq. (10.12), from the 1968 Kansas experiment. From Businger *et al.* (1971).

"Scaling" of Surface Layer data over a variety of stability conditions

(Wyngaard)

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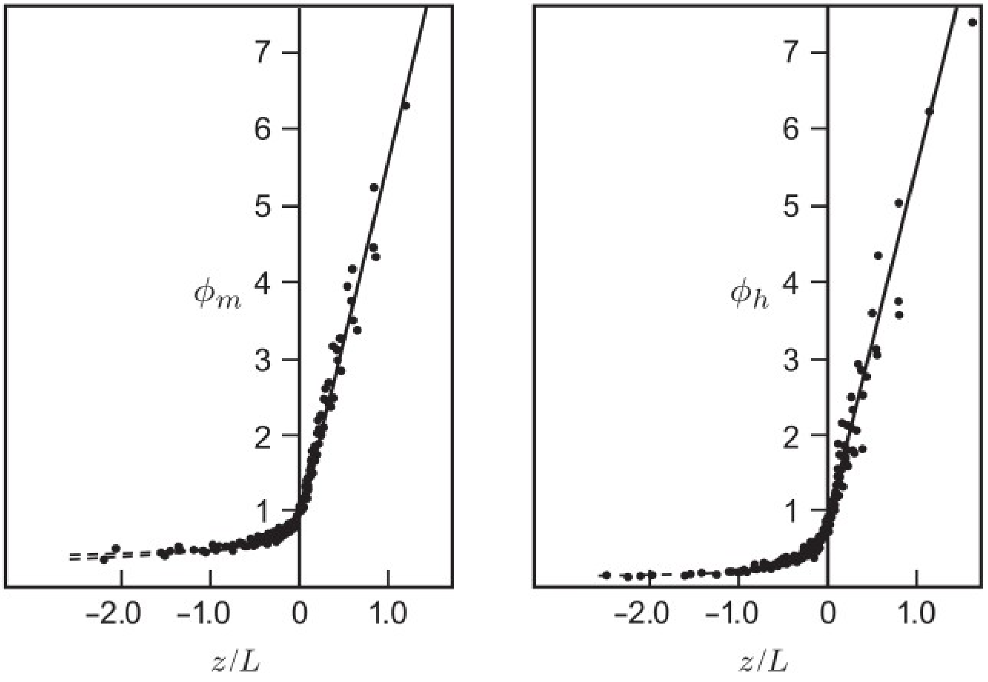

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"Scaling" of Surface Layer data over a variety of stability conditions

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"Curve Fit" equations for different portions of these universal curves are the Surface Layer Parameterizations

(Stull, pg. 355+)

4. Surface Layer Similarity Theory

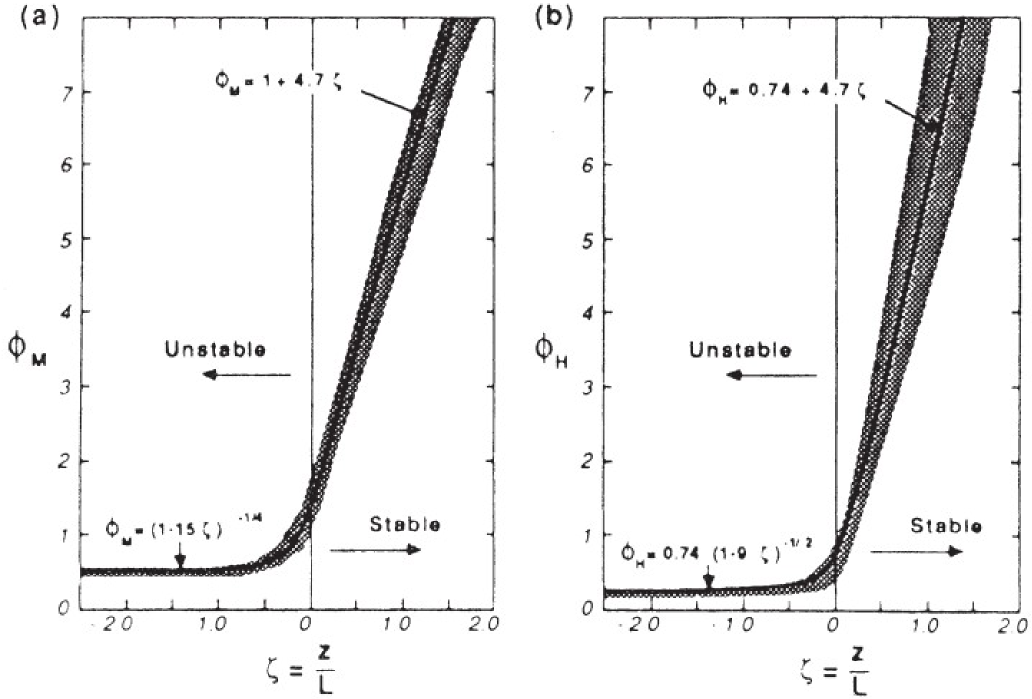

Fig. 9.9 (a) Range of dimensionless wind shear observations in the surface layer, plotted with interpolation formulas. (b) Range of dimensionless temperature gradient observations in the surface layer, plotted with interpolation formulas. After Businger, et al. (1971).

$$\phi_M = \begin{cases} 1 + \left(\dfrac{4.7\,z}{L}\right) & \text{for } \dfrac{z}{L} > 0 \text{ (stable)} \\ 1 & \text{for } \dfrac{z}{L} = 0 \text{ (neutral)} \\ \left[1 - \left(\dfrac{15z}{L}\right)\right]^{-1/4} & \text{for } \dfrac{z}{L} < 0 \text{ (unstable)} \end{cases}$$

$$\phi_H = \begin{cases} \dfrac{K_m}{K_H} + \dfrac{4.7\,z}{L} & \text{for } \dfrac{z}{L} > 0 \text{ (stable)} \\ \dfrac{K_m}{K_H} & \text{for } \dfrac{z}{L} = 0 \text{ (neutral)} \\ \dfrac{K_m}{K_H}\left[1 - \dfrac{9\,z}{L}\right]^{-1/4} & \text{for } \dfrac{z}{L} < 0 \text{ (unstable)} \end{cases}$$

5. Prandtl's concept of a mixing length

$$\tau_{viscous} = \nu \frac{\partial U}{\partial z}$$

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$$K_m \propto U_s L_s$$

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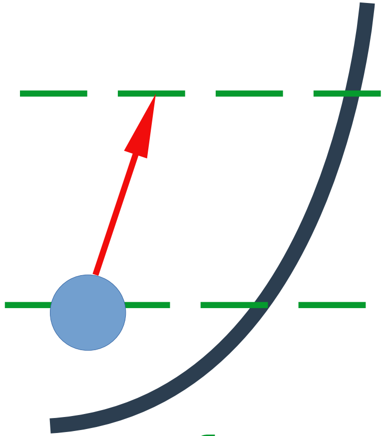

Prandtl’s concept of mixing length

ν is property of the “fluid” and K_m is property of the “flow”

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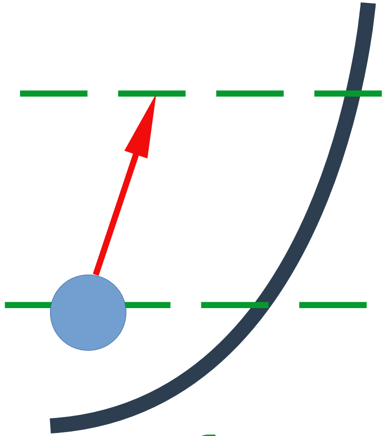
$$-\overline{u'w'} = U_*^2$$



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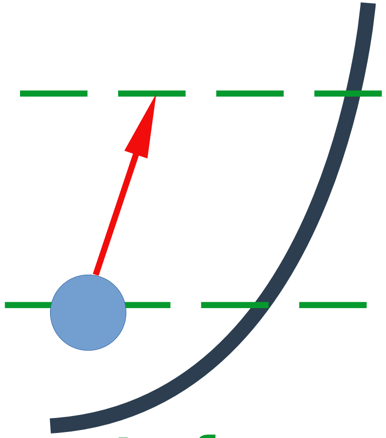

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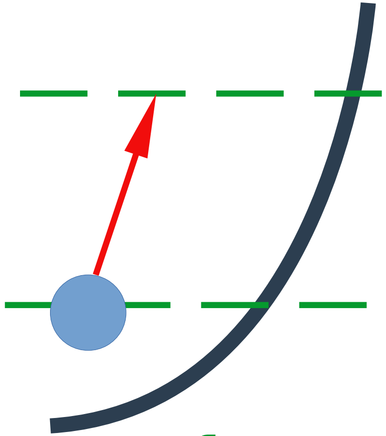

Prandtl's concept of mixing length

$$-\overline{u'w'} = U_*^2$$

$$\frac{\partial U}{\partial z} = \frac{U_*}{\kappa z}$$

$$K_m = \kappa^2 z^2 \frac{\partial U}{\partial z}$$

ν is property of the "fluid" and K_m is property of the "flow"

6. Turbulence Closures or Parameterizations

Page 197, Chapter 6 from Stull